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we may take either  $y_r = -a$ , whence  $x = (b+1)/10$ ; or  $y_r = b$ ,  $x = (-a+1)/10$ . Thus solutions go in pairs, the  $x$  of either solution being obtained by adding 1 to the  $y_r$  of the other and dividing by 10. There are, then, always an even number of solutions, and always at least two, viz., those where  $y_r = -1$ ,  $x = 10^{r-1}(n+1)$ ;  $y_r = 10^r(n+1) - 1$ ,  $x = 0$ .

Finally it may be noted that the more general equation

$$ax^r - bxy + y - c = 0$$

may be solved by similar methods. Here we should have to pick out such divisors of  $b^r c - a$  as are  $\equiv \pm 1 \pmod{b}$ . It may be interesting to apply the method to an example. Let us solve

$$x^4 - 10xy - 22 + y = 0.$$

We have to solve

$$(10x - 1)y_4 = -219999 = -3 \times 13 \times 5641$$

(5641 prime). The values of  $y_4$  with the corresponding solutions of our equation are:

$$\begin{aligned} y_4 = -5641, \quad x = 4, \quad y = 6; \quad y_4 = -1, \quad x = 22000 \text{ (} y \text{ a very large number);} \\ y_4 = 39, \quad x = -564, \quad y = -17937434; \quad y_4 = 219999, \quad x = 0, \quad y = 22. \end{aligned}$$

#### 247 (Number Theory) [June, 1916]. Proposed by NORMAN ANNING, Chilliwack, B. C.

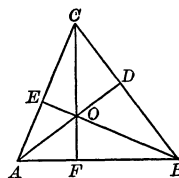
To dissect the triangle whose sides are 52, 56, 60 into three Heronian triangles by lines drawn from the vertices to a point within.

The word Heronian is used in the sense of the German Heronische (Wertheim, *Anfangsgründe d. Zahlenlehre*, p. 140) to describe a triangle whose sides and area are integral.

#### SOLUTION BY FRANK IRWIN, University of California.

The orthocenter,  $O$ , may be taken as the required point. Let  $ABC$  be the triangle with  $a = 60$ ,  $b = 52$ ,  $c = 56$ ; and let the feet of the perpendiculars from  $A, B, C$  on the opposite sides be  $D, E, F$ , respectively. Then the various lines in the figure, calculated as indicated below, are:  $BD = 168/5$ ,  $DC = 132/5$ ,  $CE = 396/13$ ,  $EA = 280/13$ ,  $AF = 20$ ,  $FB = 36$ ;  $AO = 25$ ,  $BO = 39$ ,  $CO = 33$ . Finally, area  $BOC = 594$ , area  $COA = 330$ , and area  $AOB = 420$ ; so that the sides and areas of these three triangles are integral, as asserted.

The explanation of these facts depends on the following proposition: If the sides and area of the triangle  $ABC$  are rational, the same is true of the triangles  $BOC, COA, AOB$ . (Then by multiplying the dimensions of the figure by a suitable integer everything can be made integral.) For the three altitudes are rational, as also the radius  $r$  of the inscribed circle (since  $rs = \text{area}$ ). Thus  $\tan A/2$  is rational, and so, then, are  $\cos^2 A/2$  and  $\cos A$ . Therefore,  $AF = b \cos A$  is rational, and similarly,  $FB, BD$ , etc. Then the triangle  $AOF$  is rational (that is, has rational sides), since one of its sides,  $AF$ , is rational, and it is similar to the rational triangle  $ABD$ .



#### 2678 [February, 1918]. Problem proposed by C. F. GUMMER, Queen's University, Canada.

Find necessary and sufficient conditions that the roots of the equation  $x^{n+1} + a_1x^n + a_2x^{n-1} + \dots + a_{n+1} = 0$  may be all real and separated by the roots of  $x^n + b_1x^{n-1} + b_2x^{n-2} + \dots + b_n = 0$ .

#### SOLUTION BY THE PROPOSER.

Consider the equations

$$\begin{aligned} (1) \quad f(x) &\equiv x^{n+1} + a_1x^n + \dots &= 0, \\ (2) \quad g(x) &\equiv x^n + b_1x^{n-1} + \dots &= 0, \\ (3) \quad R_1(x) &\equiv c_0x^{n-p} + c_1x^{n-p-1} + \dots &= 0, \\ (4) \quad R_2(x) &\equiv d_0x^{n-p-q} + d_1x^{n-p-q-1} + \dots &= 0, \end{aligned}$$

where  $R_1(x)$  is the remainder with sign changed on dividing  $f(x)$  by  $g(x)$ ,  $R_2(x)$  has the same relation to  $g(x)$  and  $R_1(x)$ , etc.